

Viscosity



Viscosity

Low Viscosity = More Flow

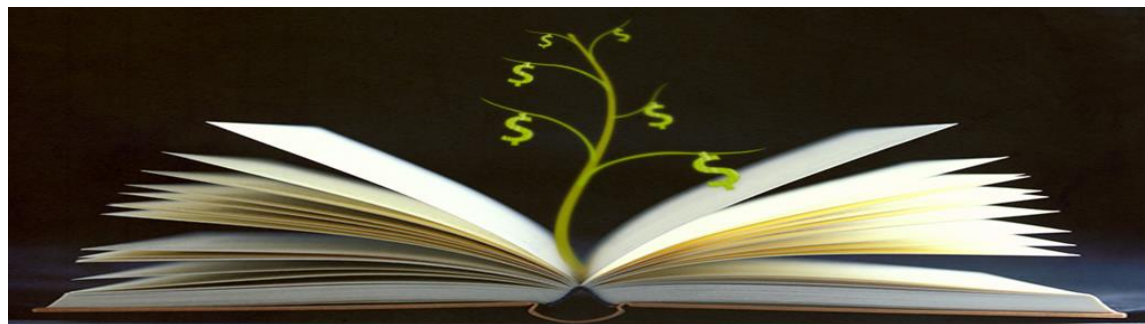


High Viscosity = Less Flow



Syllabus to be taught

- Streamline and turbulent motion, coefficient of viscosity, critical velocity, Reynold's number, Poiseuille's equation (derivation), Stokes law (derivation from dimensional formula), terminal velocity, factors affecting viscosity of a liquid (qualitative), Applications. Problems



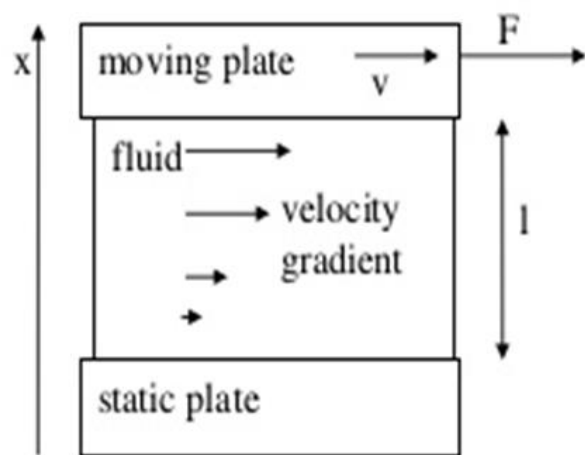
Viscosity

Have you ever noticed that some liquids like water flow very rapidly while some others like castor oil do not flow fast?

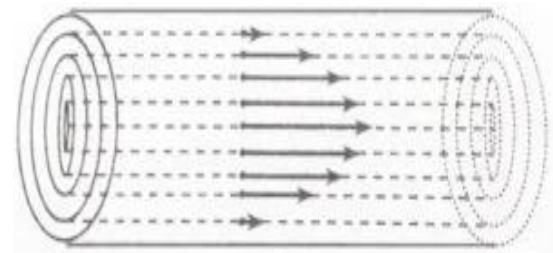
- Why is it so? Didn't that question occur to you yet?
- Well, if it did, we have the answer to it!
- This is the concept of Viscosity.
- In this chapter, we will study all about the topic and look at the laws and examples of the same.

Viscosity

- ▶ The liquid like water, kerosene flow easily.
- ▶ But honey, castor oil can't flow easily.
- ▶ For constant speed of flow we have to apply pressure difference.
- ▶ So, we note that is some force opposing speed of flow.
- ▶ It is due to viscosity.



Viscosity



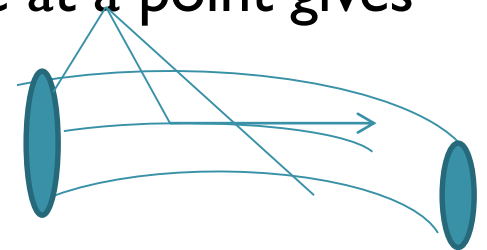
- It is the internal resistance to flow possessed by a liquid. The liquids which flow slowly, have high internal resistance. This is because of the strong intermolecular forces. Therefore, these liquids are more viscous and have high viscosity.
- The liquids which flow rapidly have a low internal resistance. This is because of the weak intermolecular forces. Hence, they are less viscous or have low viscosity.
- The retarding forces between the layers of the moving liquid are viscous forces
- **The property of liquid due to which liquid opposes relative motion between its different layer is called viscosity**

Viscosity

- In case of low fluid velocities, the fluid will flow without any sort of lateral mixing because of lack of turbulent velocity fluctuations.
- The fluid particles tend to follow a particular order where the movement or motion of fluid particles will be on the basis of particles flowing in a straight line parallel to the pipe wall. The movement happens in a way that the adjacent layers of the fluid will smoothly slide past each other.

Stream line flow

- Streamline flow in case of fluids is referred to as the type of flow where the fluids flow in separate layers without mixing or disruption occurring in between the layers at a particular point.
- The velocity of each fluid particle flowing will remain constant with time in streamline flow.
- Example: slowly and quietly flowing river, wind blowing smoothly across a surface
- Slowly flowing water in a surface
- The path along which a particle moves in a stream line is called stream line tangent to a stream line at a point gives the direction of motion
- Of all particle passing that point



Turbulent flow

- When the velocity flow of a liquid is greater than certain value the flow becomes turbulent
- In a fluid motion if the velocity at a point does not remain same then fluid motion is called turbulent
- Example: the motion of water in a high water fall, hurricane, a fast moving river

Critical velocity and Raynold's number

- If the average velocity of flow of liquid is below a certain value. The motion is stream lined , but if it exceeds this value,
- The motion becomes turbulent marked by the formation of eddies
- This limiting value of the average velocity
- Of flow is called critical velocity and marks the transition from streamlined motion in to turbulent motion

Critical velocity and Raynold's number

- Reylold showed experimentally that critical velocity v_0 is related to the density d of the liquid ,its viscosity ' η ' and the dimension 'r(radius)' of the channel through which the liquid is flowing
- By the relation $v_0 = \frac{K\eta}{rd}$ where K is the Reynolds number and for narrow tubes its about 1000

Critical velocity and Raynold's number

- For average velocity well below critical value, the rate of flow is independent of the density of the liquid but velocity higher than critical velocity the rate of flow depends mainly on the density and becomes practically independent of viscosity

Comparison between Stream line and Turbulant motion

Stream line flow

- The velocity of streamline flow is less than critical velocity
- Streamline flow is laminar
- Streamline is the path followed by the fluid particle are same
- Water flowing slowly in a pipe and motion of spermatozoa are examples of streamline flow

Turbulent flow

- the liquid moves with a velocity greater than the critical velocity of the liquid.
- turbulent flow is non-laminar.
- velocity of all the particles won't be same across all the points in the space.
- Motion of water flowing in a river and motion of blood in arteries are examples of turbulent flow.

Coefficient of viscosity η

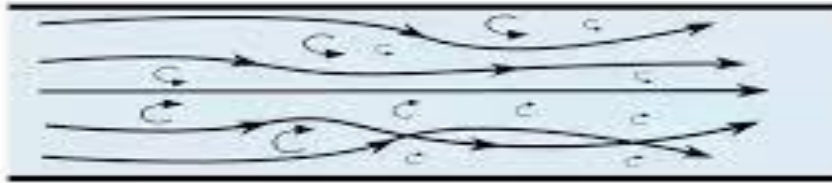
- Viscosity is the force of friction which one part of the liquid offers to another part of the liquid. The force of friction f between two layers each having area A sq cm, separated by a distance dx cm, and having a velocity difference of dv cm/sec, is given by:
- $f \propto A (dv / dx)$
- $f = \eta A (dv/dx)$
- where η is a constant known as the coefficient of viscosity and dv/dx is called velocity gradient. If $dx = 1$, $A = 1$ sq cm; $dv = 1$ cm/sec, then $f = \eta$. Hence the coefficient of viscosity may be defined as the force of friction required to maintain a velocity difference of 1 cm/sec between two parallel layers, 1 cm apart and each having an area of 1 sq cm. **SI unit is Ns/m^2**

Viscosity

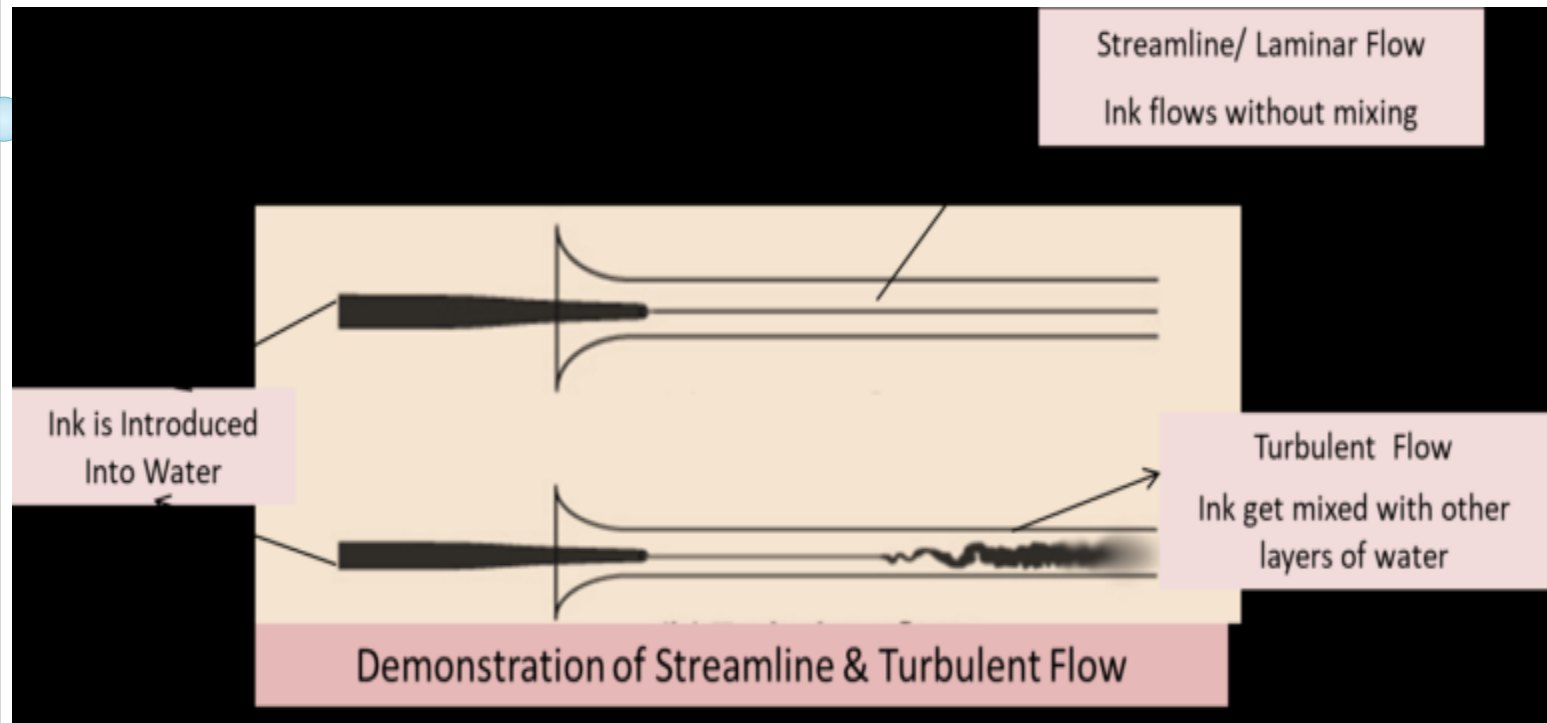
laminar flow



turbulent flow

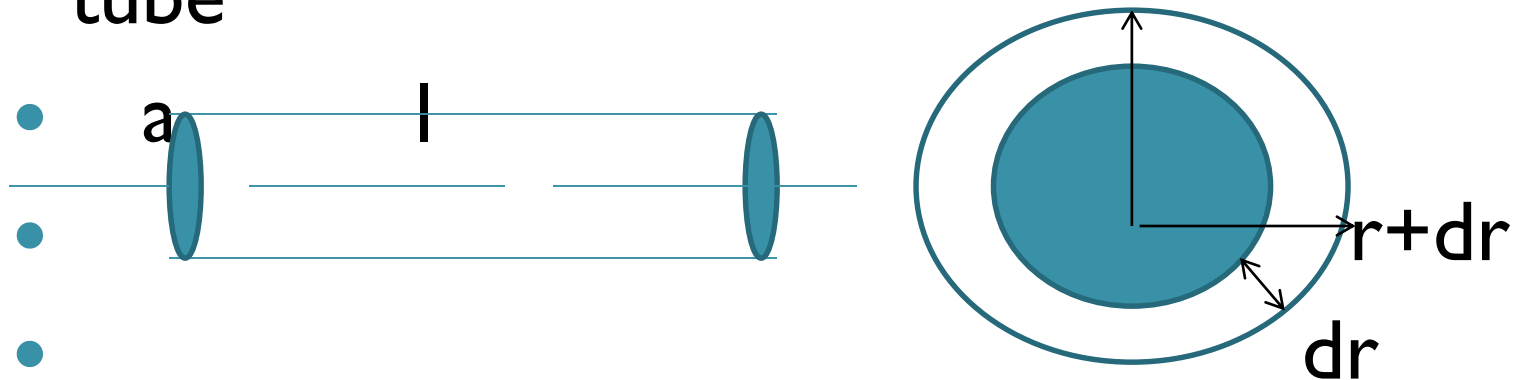


Viscosity



Poiseuille's formula

- Consider a flow of liquid through a horizontal capillary tube under a constant pressure difference p at its ends let l be the length a be the radius of the tube



Poiseuille's formula

- In this case following assumptions are made
 1. The flow is steady
 2. The tube is horizontal such that there is no gravity influence on the flow
 3. The pressure over any normal cross section is constant
 4. Liquid layer which is in contact with the walls of the tube is at rest

Poiseuille's formula

- Now take a cross section of the tube
- Consider a cylindrical layer of the liquid
- Co axial with the tube and having inner radius r and outer radius $r+dr$
- Let v and $v+dv$ are the velocities of liquid
- At inner and outer surface of the this cylindrical layer respectively then $\frac{dv}{dr}$ is the velocity gradient

Poiseuille's formula

- Surface area of the cylinder = $2\pi r l$
- Viscous force between
- the layers $F = -\eta 2\pi r l \frac{dv}{dr}$
- Forward force on the
- cylindrical layer = $p * \pi r^2$
- When motion is steady (no acceleration of the fluid)
- Viscous force between the layers = forward force on the cylindrical layer

Poiseuille's formula

- $\eta 2\pi r l \frac{dv}{dr} = p^* \pi r^2$
- $dv = -\frac{p}{2\eta l} * r dr$
- $\int v = -\frac{p}{2\eta l} * \frac{a^2}{2} + c$
- Constant of integration c is determined using boundary condition $v=0$, where $r=a$
- $0 = -\frac{p}{2\eta l} * \frac{a^2}{2} + c$

Poiseuille's formula

- Therefore $v = \frac{p}{4\eta l}(a^2 - r^2)$ this is the velocity of a layer at a distance r from the axis
- Area of cross section of the cylindrical layer of thickness $dr = 2\pi r \cdot dr$
- Volume of the liquid flowing per second through this area $dv = v \cdot 2\pi r \cdot dr$
- Volume of the liquid flowing per second
- $V = \int_0^a dv$

Poiseuille's formula

- $V = \int_0^a v * 2\pi r dr = \int_0^a \frac{p}{4\eta l} (a^2 - r^2) * 2\pi r dr$
- $= \frac{\pi p}{2\eta l} \int_0^a (a^2 r - r^3) dr = \frac{\pi p}{2\eta l} \left[a^2 \frac{r^2}{2} - \frac{r^4}{4} \right]$
- $\frac{\pi p}{2\eta l} \left(\frac{a^4}{2} - \frac{a^4}{4} \right) = \frac{\pi p a^4}{8\eta l}$
- $V = \frac{\pi p a^4}{8\eta l}$ rate of flow of liquid (volume of liquid flowing per second)
- $\eta = \frac{\pi p a^4}{8Vl}$ this is poiseuille formula
- By knowing p, a, l and v the co-efficient of viscosity can be calculated

Stokes law

- According to stokes for a small ball falling through a viscous medium the viscous force F acting on the ball depends on
- Radius r of the ball
- Terminal velocity v of the ball and
- Co-efficient of viscosity η of the liquid
- The viscous force F is determined by the method of dimension as follows

Stokes law

- $F \propto r^a v^b \eta^c$
- Where K is a dimension less constant taking dimensions on both side
- Or $[F] = k[r^a][v^b][\eta^c]$
- Substituting the dimensions of the physical quantities
- $M^1 L^1 T^{-2} = K(L)^a (LT^{-1})^b (M^1 L^{-1} T^{-1})^c$
- $= k * L^a * L^b T^{-b} * M^1 L^{-c} T^{-c}$

Stokes law

- Equating the dimension on both side
- $l=C$, $l=a+b-c$ and $-2=-b-c$
- $l=a+b-l$,and $2=b+l$
- $2=a+l$ and $l=b$
- $a=l$
- $F=r\nu\eta$
- Stokes showed that $k=6\pi$
- Therefore $F=6\pi r\nu\eta$ stokes law

Terminal velocity

- When small steel ball is kept slowly on the surface of liquid the ball is initially accelerated due to its weight, The motion of the ball is opposed by Viscous force and upward thrust due to buoyancy of the liquid this viscous force increases as the velocity of ball increases when sum of viscous force and upward thrust is equal to the weight then ball moves with a constant velocity called terminal velocity

Terminal velocity

- “The constant velocity attained by body due to viscous force while moving in a fluid is called terminal velocity”
- during the down ward motion of the ball forces acting on it are
- Weight W of the ball acting downward
- Force of buoyancy F_B acting upward
- Viscous force F acting upward

Terminal velocity

- Initially there is a net downward force therefore ball is accelerated if the ball is small it soon attains a constant velocity called terminal velocity
- now weight W of the ball = viscous force F + force of buoyancy F_B
- Weight of the ball $W = mg = \frac{4}{3}\pi r^3 \rho g$
- When $r =$ radius of the ball $\rho =$ *density of the ball*

Terminal velocity

- $F = 6\pi r v \eta$
- Where $v =$ terminal velocity of the ball
- $\eta =$ coefficient of viscosity of the liquid
- Force of buoyancy $F_B =$ weight of the liquid displaced $= \frac{4}{3} \pi r^3 d g$
- Where $d =$ density of liquid

Terminal velocity

- Now relation I can be written as
- $\frac{4}{3}\pi r^3 \rho g = 6\pi r v \eta + \frac{4}{3}\pi r^3 d g$
- $\frac{4}{3}\pi(\rho - d)g = 6\pi r v \eta$
- $V = \frac{2}{9} \frac{r^2(\rho - d)g}{\eta}$ terminal velocity
- Rain drop fall in air with a constant velocity due to viscosity of air

Variation of viscosity with temperature

- Viscosity of liquids decreases with temperature
- Due to rise of temperature kinetic energy of molecules increases and cohesive force decreases
- Velocity of gases increases with temperature
- In the case of gases Viscosity is due to diffusion of molecules between the layers

Variation of viscosity with temperature

- With temperature diffusion rate increases
- Which creates more friction

Applications of viscosity

- The importance of viscosity can be understood from the following examples.
- I) The oil used as a lubricant for heavy machinery parts should have a high viscous coefficient. To select a suitable lubricant, we should know its viscosity and how it varies with temperature [Note: As temperature increases, the viscosity of the liquid decreases]. Also, it helps to choose oils with low viscosity used in car engines (light machinery).

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Applications of viscosity

2) The highly viscous liquid is used to damp the motion of some instruments and is used as brake oil in hydraulic brakes.

3) Blood circulation through arteries and veins depends upon the viscosity of fluids.

4) Millikan conducted the oil drop experiment to determine the charge of an electron. He used the knowledge of viscosity to determine the charge.